Completing "Truth Table Choice" Problems

Here we explain how to complete "truth table choice" problems, which we will be doing in WebCT assignments.

Problem: For the following argument, (i) **list each sentence** that shows up in the process of building a truth table for the argument; then (ii) **state**, **for each sentence** you've listed, **which** of the truth table choices **is the truth table for that sentence**; and finally (iii) state whether the argument is **valid** or **invalid**.

Argument:

1. $\sim(P \wedge Q)$
2. P

$\therefore\ \sim Q$

Truth Tables Choices:

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
0	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1
1	0	0	1	1	0	1	0
1	0	0	0	1	1	0	0

Discussion: To complete this problem, we must first build a truth for each sentence in the argument (the two premises, and the conclusion).

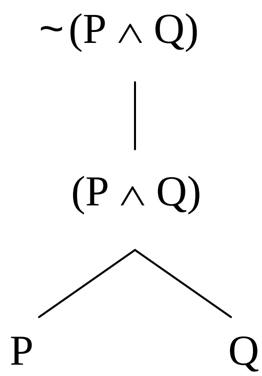
We can see, first of all, that all the sentences are built up out of sentence letters "P" and "Q". So we know our truth table for this argument will start with these sentence letters.

P	**Q**

And we give these two sentence letters values in the usual manner.

P	**Q**
1	1
1	0
0	1
0	0

The first premise, “~(P ∧ Q),” was built up in the following manner.



The truth table for Premise (1) will follow these steps. “P” and “Q” are already in the truth table; so to finish the truth table for this premise, we just need to add the remaining two steps – “(P ∧ Q)” and “~(P ∧ Q)”.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**
1	1		
1	0		
0	1		
0	0		

The second premise is just “P,” which is already in the truth table.

The conclusion, "~Q," is not yet in the truth table; so we need to add this sentence.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1			
1	0			
0	1			
0	0			

Having listed the required sentences for this argument, across the top of the truth table, we only need to add the truth values for each sentence, to complete the table.

"(P ∧ Q)" is a conjunction, and so follows the conjunction rule. A conjunction is only truth when both of the parts are true.

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

And that means that "(P ∧ Q)" is true in the first valuation,

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	**1**		
1	0			
0	1			
0	0			

and false in Valuations (2) through (4).

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1		
1	0	**0**		
0	1	**0**		
0	0	**0**		

“~(P $\wedge$ Q)” is the negation of “(P $\wedge$ Q)”. Being a negation, it will follow the negation rule.

●	~●
1	0
0	1

So when the original sentence, “(P $\wedge$ Q),” is true (Valuation 1), its negation will be false,

P	**Q**	**(P $\wedge$ Q)**	**~(P $\wedge$ Q)**	**~Q**
1	1	1	**0**	
1	0	0		
0	1	0		
0	0	0		

and when the original sentence is false (Valuations 2 through 4), its negation will be true.

P	**Q**	**(P $\wedge$ Q)**	**~(P $\wedge$ Q)**	**~Q**
1	1	1	0	
1	0	0	**1**	
0	1	0	**1**	
0	0	0	**1**	

“~Q” is also a negation, and so will also follow the negation rule: when “Q” is true (Valuations 1 and 3), “~Q” will be false.

P	**Q**	**(P $\wedge$ Q)**	**~(P $\wedge$ Q)**	**~Q**
1	**1**	1	0	**0**
1	0	0	1	
0	**1**	0	1	**0**
0	0	0	1	

And when “Q” is false (Valuations 3 and 4), “~Q” will be true.

P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	0
1	**0**	0	1	**1**
0	1	0	1	0
0	**0**	0	1	**1**

With the truth table for this argument completed, we are now in a position to complete the problem. We needed to ***first*** make a list of each of the sentences that show up in the truth table for this argument. That’s easy: these are just the sentences listed across the top of the truth table. So our list will be as follows.

<u>Sentences</u>:

P
Q
(P ∧ Q)
~(P ∧ Q)
~Q

Second, we needed to state, for each of these sentences, which of the numbered “truth table choices” is the right table for that sentence.

Truth Table Choices:

(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
0	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1
1	0	0	1	1	0	1	0
1	0	0	0	1	1	0	0

Checking with our completed truth table for this argument, we see that “**P**” takes truth table choice **(viii)**; “**Q**” takes **(iv)**; “**(P $\wedge$ Q)**” takes **(ii)**; “**~(P $\wedge$ Q)**” takes **(v)**; and “**~Q**” takes **(vi)**.

So we put the matching truth table number next to each sentence in our list.

P	**Q**	**(P $\wedge$ Q)**	**~(P $\wedge$ Q)**	**~Q**
1	1	1	0	0
1	0	0	1	1
0	1	0	1	0
0	0	0	1	1

Truth Table Choices:

(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
0	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1
1	0	0	1	1	0	1	0
1	0	0	0	1	1	0	0

Sentences, and Matching Truth Tables:

P: (viii)
Q: (iv)
(P $\wedge$ Q): (ii)
~(P $\wedge$ Q): (v)
~Q: (vi)

Finally, we needed to say – based on our truth table for the argument – whether the argument is valid or invalid. We decide this by first picking out the valuations where all of the premises are true. Only Valuation (2) makes all the premises true.

1			**2**	**∴**
P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	0
1	0	0	**1**	1
0		0	1	0
0	0	0	1	1

And in that valuation, the conclusion is also true.

1			**2**	**∴**
P	**Q**	**(P ∧ Q)**	**~(P ∧ Q)**	**~Q**
1	1	1	0	0
1	0	0	**1**	**1**
0	1	0	1	0
0	0	0	1	1

Since every situation making (all) the premises true also makes the conclusion true, this argument meets our definition of a valid argument: **the argument is valid**. Putting in our verdict on the argument's validity completes the assignment.

Final Answer:

Sentences, and Matching Truth Tables:

P: (viii)
Q: (iv)
(P ∧ Q): (ii)
~(P ∧ Q): (v)
~Q: (vi)

Verdict: Argument is Valid

This is how we will do truth table problems in WebCT. As this example illustrates, you ***will*** need to do the truth table for the argument, in order to answer the questions properly. But by doing the problem in this list, and "truth table choice" fashion, we avoid trying to type out truth tables in the text boxes of WebCT assignment (which would be a big mess for everyone involved).